III- SIGNAL SENSING

a. Spectrum Analysis

The aim of signal sensing and spectrum analysis is to define the occupancy of some slices of the spectrum. In fact, we have to discriminate two cases summerized by the following hypotheses

$$\begin{aligned} &H_0 : \boldsymbol{y}[k]=\boldsymbol{n}[k],\ k=1,\ \ldots,\ \mathrm{N} \\ &H_1 : \boldsymbol{y}[k]=\boldsymbol{x}[n] + \boldsymbol{n}[k], \end{aligned} \qquad \textbf{(1)}$$

with $\boldsymbol{x}[k]$ the primary user's signal, $\boldsymbol{n}[k]$ the additive noise, zero-mean, white, circularly symmetric complex Gaussian, as $\boldsymbol{n} \sim \mathcal{N}(0,\sigma^2\mathrm{I})$, where σ^2 is its variance, and $\boldsymbol{y}[k]$ the vector containing elements corresponding to the different received signals by the antennas at the k time.

For simplicity of notation, we will note $\boldsymbol{y}_\mathrm{m}[k]$ as the observation of the $\boldsymbol{y}$ vector for the m^th user (or antenna) at time k.

Hence, elements of $\boldsymbol{y}$ are time sequences of M users or antennas, and each sequence is made up of L scalar elements. Then $\boldsymbol{y}$ may be rewritten as a matrix of (M x LN) elements that contains the whole observations for all the users

$$\boldsymbol{y} = \begin{matrix} \boldsymbol{y}_1[1]^T & \cdots & \boldsymbol{y}_1[N]^T \\ \vdots & \ddots & \vdots \\ \boldsymbol{y}_\mathrm{M}[1]^T & \cdots & \boldsymbol{y}_\mathrm{M}[N]^T \end{matrix} \qquad \textbf{(2)}$$

where each element $\boldsymbol{y}_\mathrm{m}[k]$ is a vector of length L. To simplify comprehension of the next parts, we note $\boldsymbol{Y}_\mathrm{i}$ the i^th row of $\boldsymbol{y}$. Obviously, the same structure can be built for $\boldsymbol{x}$ and $\boldsymbol{n}$, so that the hypothesis **(1)** could be rewritten as

$$\begin{aligned} &H_0 : \boldsymbol{y}=\boldsymbol{n}, \\ &H_1 : \boldsymbol{y}=\boldsymbol{x} + \boldsymbol{n}. \end{aligned}$$

b. Presence Detector

Now that the hypotheses are taken, the goal is to find a way to detect if the system is in one case or in the other one. So that the state of communication is known, we have to chose a statistic test depending upon the mixed signal $\boldsymbol{y}$. This test will be noted $\Lambda(\boldsymbol{y})$. Then, thanks to a comparison with a threshold η, it is possible to access the information

$$\begin{aligned} &H_0 : \Lambda(\boldsymbol{y}) < \eta \\ &H_1 : \Lambda(\boldsymbol{y}) \geq \eta. \end{aligned} \qquad \textbf{(3)}$$

The statistic test depends upon the chosen detector. To judge the efficiency of a detector, the best way is to draw the curve of its receiver operating characteristics (ROC). This curve ties down the detection probability, noted $\mathrm{P_D}$, and the false alarm detection probability $\mathrm{P_{FA}}$. In fact, the ROC curve is the representation of the function $\mathrm{P_D}=\mathrm{f}(\mathrm{P_{FA}})$

$$\begin{aligned} &\mathrm{P_D} = \mathrm{P}(\Lambda(\boldsymbol{y}) \geq \eta \mid \mathrm{H_1}) \\ &\mathrm{P_{FA}} = \mathrm{P}(\Lambda(\boldsymbol{y}) \geq \eta \mid \mathrm{H_0}) \end{aligned} \qquad \textbf{(4)}$$

In proportion to the threshold η probabilities of detection varies, then the performance point on the ROC curve glides.

In the present situation, the purpose is to maximize $\mathrm{P_D}$. On account of this, we have to chose the right statistic test $\Lambda(\boldsymbol{y})$ and the corresponding threshold η, with a chosen limited noise-free $\mathrm{P_{FA}}$.

c. Constant False Alarm Rate Criterion

As the signal detection has to work with no *a priori* of signals power, the detector has to allow a constant false alarm rate (CFAR). The willingness of a constant behaviour is due to the fact that $\Lambda(\boldsymbol{y})$, η, $\mathrm{P_D}$ and $\mathrm{P_{FA}}$ are bound up in each others **(4)**. If the $\mathrm{P_{FA}}$ varies according to power of signals, the threshold would have to be dynamical. As it is unacceptable in this situation, the detector must have the CFAR characteristic.

d. Energy Detector

This kind of detector works by comparing the energy of each $\boldsymbol{Y}_i$ to an adapted threshold.

In fact, ($\boldsymbol{Y}_i | H_0$) and ($\boldsymbol{Y}_i | H_1$) should not have the same level of energy. Thus, it is not necessary to know the structure of the analyzed signal, but only calculate the energy.

i. Theoretical case – Khi²

As the structure of signal does not influence the response of the detector, the best way to appreciate its behaviour is to test it with different common deterministic distributions.

In this part, we suppose that $\boldsymbol{x}$ is a zero-mean circularly symmetric complex Gaussian, with γ^2 as a variance, then $\boldsymbol{x} \sim \mathcal{N}(0, \gamma^2 I)$. Thus, $\boldsymbol{Y}_i | H_1 \sim \mathcal{N}(0, (\sigma^2 + \gamma^2) I)$ and $\boldsymbol{Y}_i | H_0 \sim \mathcal{N}(0, (\sigma^2 I))$.

The statistic test $\Lambda(\boldsymbol{Y}_i)$ can now be written as

$$\Lambda(\boldsymbol{Y}_i) = \frac{\|\boldsymbol{Y}_i\|}{\sigma^2} = \frac{\sum_{j=1}^{LN} |\boldsymbol{Y}_{ij}|^2}{\sigma^2},$$

$$H_0 : \Lambda(\boldsymbol{Y}_i) < \eta,$$

$$H_1 : \Lambda(\boldsymbol{Y}_i) \geq \eta, \qquad \textbf{(5)}$$

so that **(4)** can be rewritten as

$$P_D = \mathrm{P}(\Lambda(\boldsymbol{Y}_i) \geq \eta \mid H_1) \qquad \textbf{(6)}$$

$$= 1 - F_{\chi^2{}_{NL}}\left(\frac{2\eta}{\sigma^2 + \gamma^2}\right)$$

$$= 1 - F_{\chi^2{}_{NL}}\left(\frac{F_{\chi^2{}_{NL}}{}^{-1}(1 - P_{FA})}{1 + \frac{\gamma^2}{\sigma^2}}\right).$$

Considering last results **(7)**, the knowledge of the khi distribution, variances, and a fixed PFA allow to fill a vector of theoretical thresholds. In fact, for each P_{FA} into the interval [0,1] corresponds a theoretical threshold.

This way, the theoretical ROC has been drawn *figure 1*.

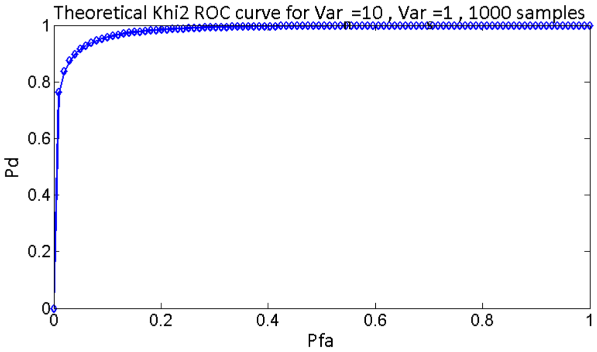

FIGURE 1 : THEORETICAL ROC CURVE, CONSIDERING A THOUSAND SAMPLES OF A SIGNAL, WHOSE VARIANCE IS 1. NOISE VARIANCE IS 10.

ii. Actual Case – 16-QAM

Now the thresholds are created, the 16-QAM signal Yi can be tested. To draw the ROC of this case the algorithm summarized by the flow diagram figure 2, tests each Λ(Yi) with the theoretical thresholds.

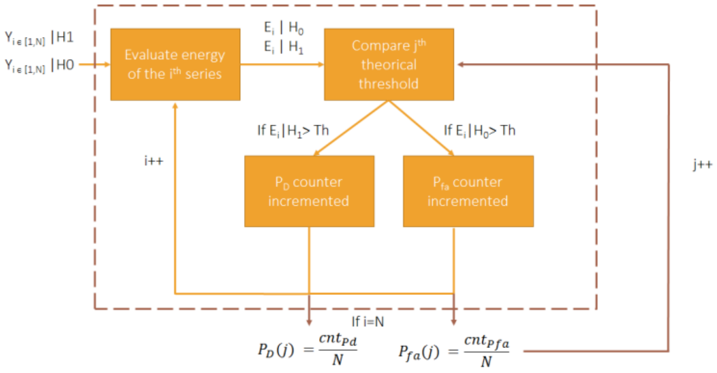
$Y_{i \in [1,N]}$ |H1
$Y_{i \in [1,N]}$ |H0
Evaluate energy of the ith series
E_i | H_0
E_i | H_1
Compare jth theorical threshold
If $E_i|H_1$> Th
If $E_i|H_0$> Th
i++
j++
P_D counter incremented
P_{fa} counter incremented
If i=N
$P_D(j) = \frac{cnt_{Pd}}{N}$
$P_{fa}(j) = \frac{cnt_{Pfa}}{N}$

FIGURE 2 : FLOW DIAGRAM OF THE ROC DRAWING ALGORITHM, IN THE 16-QAM CASE.

The tests results increase the probability of detection P_D or the probability of false-alarm P_{FA}, so that the ROC can be drawn. For several possible noise variances, the *figure a* (annex) highlights the curves for different SNR and frame sizes.